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Meson–Lepton Interaction in the Spinor Strong Interaction Theory

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The recently developed spinor strong interaction theory, which successfully accounts for linear confinement and classification of mesons as well as unmixed meson spectra, is applied to semileptonic decays of the π, K, D, and B mesons. These mesons themselves generate mass M_w for the mediating gauge boson; no Higgs boson is needed. The theory is also applied to purely leptonic interactions. It is shown that the results of the standard electroweak model can be taken over with the Higgs boson replaced by the above mesons. The Cabbibo angle ϑ_C is given by tan ϑ_C = (pion mass)/(kaon mass), in agreement with data. The pion decay constant F is essentially a ratio between two large constants introduced to make certain infinite integrals finite. M_w is also related to a similar cutoff constant.

1. INTRODUCTION

The present theory of kaon and pion decay into muons and neutrinos is phenomenological and over three decades old (Källén, 1964; Lee, 1981). It is essentially kinematical and accounts for the ratio of the rate of pion decay into muons to that into electrons. The strong interaction parts of these decays are expressed in terms of two unknown constants, the pion decay constant F and the Cabbibo angle ϑ_C, which are determined by the two observed decay rates. Thus, the theory is left with no further predictive power. In addition, the Cabbibo angle obtained differs considerably from those obtained from other data, for instance, baryon beta decay.

Recently, a strong interaction theory (Hoh, 1993; hereafter denoted by I) has been developed. Linear confinement arises naturally and no approximation intervenes in the basic covariant starting equations and the mass spectra of unmixed ground-state mesons. The predicted masses agree well with data and the meson spectra are classified accordingly (Hoh, 1996). These results

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are not obtainable in conventional models, which also predict many unseen states. Gauge invariance of the spinor strong interaction theory (Hoh, 1994a; hereafter denoted by II) accounts for the gauge boson masses without the unseen Higgs boson and resolves the so-called $U(1)$ problem. The zero mass of the photon requires the nonexistence of the pseudoscalar isosinglets $u\bar{u} + d\bar{d}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}(\eta_b)$ and forces them to become the mixed mesons η, η_c, and possibly η' which do not transform according to $U(1)$.

The purpose of this paper is to apply the spinor strong interaction theory to semileptonic decays of pseudoscalar mesons typified by K^+ and $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and to purely leptonic interactions. The main results are $\tan \vartheta_C = m_\pi/m_K$, in near agreement with data, and that the purely leptonic interaction results of the standard electroweak model without quarks, such as the decay of the muon at rest, can be taken over without the aid of the unseen Higgs bosons.

In Section 2, the action including isodoublets of II is augmented by a lepton part for the kaon decay and solutions of I are reviewed with the aid of Appendix A. These solutions are modified by introducing two large constants in Section 3 to render two infinite integrals finite. In Section 4, the decay amplitude is obtained from the first-order terms of the action dependent upon the gauge boson field W^-. Here, quantization aspects of the spinor strong interaction theory considered in II and improved upon in Appendix B are used. The W^- field as well as its mass M_W are derived from the second-order terms in the total action in Section 5. The decay rate is then obtained in Section 6 using the decay volume derived in Section 7.

Section 8 shows that the meson action of Section 2 remains unchanged after generalization to include internal functions and operators. In Section 9, the action including isotriplets of II is augmented by the same lepton part as above. It is shown that inclusion of internal operations is essential for a correct action. The pion decay rate is derived in Section 10 and differs from that of the kaon only in their masses. Comparison with data is carried out in Section 11. With the aid of M_W and the quantized Hamiltonian obtained in Appendix B, the large constants introduced in Section 3 are estimated and found to be indeed large and are therefore justifiable.

In Section 12, the action of the standard electroweak model without quarks is compared to the action of Section 2 with respect to purely leptonic interactions and extensive conclusions are drawn.

2. ACTION FOR ISODOUBLET MESON DECAY

In the standard electroweak model (see e.g., Lee, 1981) the total action consists of a gauge boson part, a quark part, a Higgs part, and a lepton part, together with their couplings. The action S_{T2} of (II 6.6) consists of the same gauge boson part and a meson part replacing the quark and Higgs parts above, but contains no lepton part.

The action employed here is basically the same as S_{T2} above complemented by the lepton part, together with their couplings. $K^+ \to \mu^+ + \nu_\mu$ will be considered as a typical semileptonic decay of isodoublet mesons. The action reads

$$S_{2ML} = S_{F2} + S_{Lr} + S_{Ll} + S_{Lm} + S_{M2} \tag{2.1}$$

Here, S_{F2} is the gauge boson action (II 6.5),

$$\begin{aligned} S_{F2} &= -\frac{1}{4}\int d^4X \left(F_C^{\alpha\beta} F_{C\alpha\beta} + \sum_{l=1}^{3} G_l^{\alpha\beta} G_{l\alpha\beta} \right) \\ F_C^{\alpha\beta} &= \partial^\alpha C^\beta - \partial^\beta C^\alpha \\ G_l^{\alpha\beta} &= \partial^\alpha W_l^\beta - \partial^\beta W_l^\alpha - \epsilon_{jkl} g W_j^\alpha W_k^\beta \end{aligned} \tag{2.2}$$

S_{Lr}, S_{Ll}, and S_{Lm} are the massless right-handed singlet, left-handed doublet, and muon mass parts, respectively, of the lepton action. In spinor form, they read

$$S_{Lr} = -\frac{i}{4}\int d^4X\, \psi_\mu^{\dot a}\left(\partial_{\dot a b} + \frac{i}{2} Yg' C_{\dot a b} \right)\psi_\mu^b + \text{c.c.} \tag{2.3a}$$

$$S_{Ll} = -\frac{i}{4}\int d^4X\, \chi_{da}^\dagger D^{a\dot b} \chi_{d\dot b} + \text{c.c.}$$

$$\chi_{d\dot b} = \begin{pmatrix} \chi_{\nu\dot b} \\ \chi_{\mu\dot b} \end{pmatrix} \tag{2.3b}$$

$$S_{Lm} = -\frac{1}{4}\int d^4X\, m_\mu(\psi_\mu^{\dot a}\chi_{\mu\dot a} + \text{c.c.}) \tag{2.3c}$$

$$D^{a\dot b} = \partial^{a\dot b} + \frac{i}{2} g\boldsymbol{\sigma}\mathbf{W}^{a\dot b} + \frac{i}{2} Yg' C^{a\dot b} \tag{2.3d}$$

where $\chi_{da}^\dagger$ is the Hermitian conjugate of $\chi_{d\dot a}$ and Y the hypercharge. The subscripts μ and ν refer to muon and neutrino, respectively, and m_μ is the muon mass. The meson action S_{M2} is given by (II 6.4),

$$S_{M2} = \int d^4X\, \mathcal{L}_{M2} =$$

$$-\int d^4X \int d^4x\, \frac{1}{2}\left\{\left(\frac{1}{2}\{[(1-a)D^{\dot b a} - \partial_x^{\dot b a}]\Psi_{\dot d}^+(X)\chi_{\dot a}^{\dot e}(x)\}\right.\right.$$

$$
\left.\left.\begin{aligned}
&\times [(aD_{f\dot{e}} + \partial_{xf\dot{e}})\Psi_d(X)\chi^{\dot{f}}_{b}(x)] \\
&+ \frac{1}{2}[(aD^{\dot{b}a} + \partial_x^{\dot{b}a})\Psi_d^+(X)\psi_a^{\dot{e}}(x)][(1-a)D_{f\dot{e}} - \partial_{xf\dot{e}}]\Psi_d(X)\psi_b^{\dot{f}}(x) + \text{h.c.}\right) \\
&+ (\phi_p - M_m^2)\Psi_d^+(X)\Psi_d(X)[\psi_b^{\dot{c}}(x)\chi_{\dot{c}}^{b}(x) + \text{h.c.}]\right\}
\end{aligned}\right. \tag{2.4}
$$

where

$$
\Psi_d(X) = \begin{pmatrix} \Psi_{d+}(X) \\ \Psi_{d0}(X) \end{pmatrix} \tag{2.5}
$$

Ψ_{d+} refers to K^+ and Ψ_{d0} to K^0. *CP* symmetry violation effects are ignored. In Appendix A, the various symbols are explained where deemed necessary and some results of I and II are reproduced. The first part of Appendix B considers the change of the normalization constant (B1) in S_{T2} to unity.

The spinor strong interaction theory is based upon manipulations of van der Waerden's spinors, the right-handed ψ^a and the left-handed $\chi_{\dot{b}}$, a, b = 1, 2. These appear naturally in the right- and left-handed actions (2.3a), (2.3b). The γ_0 and γ_5 matrices carried along in the derivations in Dirac's bispinor formalism are avoided here.

$SU(2) \times U(1)$ invariance of S_{F2} and S_{M2} has been shown in Section 6 of II. This invariance is simply extendable to S_{Lr} and S_{Ll} of (2.3) since the forms therein are already present in (2.4). S_{Lm}, however, is not invariant under $SU(2) \times U(1)$ transformations, so that a Glashow model, which disregards this noninvariance, is adopted in this respect. This inconsistency is related to the fundamental problem of the origin of lepton masses.

The terms in the action (2.1) can be grouped in powers of the small parameters g and g'. The assigned ordering is as follows:

- zeroth order: terms in S_{M2} not containing g or g'
- first order: g, g' and other first-order terms in S_{M2}, terms in S_L not containing g or g' (2.6)
- second order: g^2, gg', and g'^2 terms in S_{M2}, g and g' terms in S_L, terms in S_{F2} not containing g

To zeroth order, only S_{M2} with g and $g' \to 0$ survives, which upon variation yields the basic meson equations (A6). With (A5), the steady-state solution of these equations for ground-state pseudoscalar mesons at rest is given by, according to Section 7 of I,

$$
\begin{pmatrix} \Psi_{d+} \\ \Psi_{d0} \end{pmatrix} = \begin{pmatrix} A_{d+} \\ A_{d0} \end{pmatrix} e^{-iE_0X^0} \tag{2.7}
$$

$$
\chi^{a\dot{e}}(x) = \delta^{a\dot{e}}\chi_0(x) = \delta^{a\dot{e}}\chi_0(r)e^{i\omega_0x^0} = -\psi^{a\dot{e}}(x), \qquad r = |\mathbf{x}| \tag{2.8}
$$

Here E_0 is the mass of the meson neglecting electromagnetic correction, ω_0 is the relative energy among the quarks, and $\chi_0(r)$ is the radial meson wave function in relative space determined by (A7) and (A8). Further, (I 6.6) and (II 5.2b) read

$$a = \tfrac{1}{2} + \omega_0/E_0, \qquad e_w = 1 - 4\omega_0^2/E_0^2 \tag{2.9}$$

The first is an allowed choice and the last a definition.

Now, $A_{d+} = A_{d0} = A_d$ for given $\chi_0(r)$ is a fixed quantity related to the confinement slope β_{m0} of (I 7.6b) or

$$\phi_p(r \to \infty) = -\beta_{m0} r \tag{2.10a}$$

$$2\beta_{m0} = A_d^2 \int_0^\infty dr\, r^2 \chi_0^2(r) \tag{2.10b}$$

where only one of K^+ or K^0 is considered. Note that

$$A_d \neq 1/\sqrt{2E_0\Omega} \xrightarrow{\Omega\to\infty} 0 \tag{2.11}$$

where Ω is a large normalization volume.

3. MODIFIED MESON WAVE FUNCTIONS

Due to (2.11), the integrals over $\mathbf{X}$ in (2.4) become infinite for the zeroth-order solution (2.7) and (2.8). Likewise, the integral over the relative time x^0 is also finite. These infinities are considered in Appendix B. For this reason, I make the Ansatz

$$\begin{pmatrix} \Psi_{d+} \\ \Psi_{d0} \end{pmatrix} = \begin{pmatrix} a_{d+}^{(0)} + a_{d+}^{(1)}(X^0) \\ a_{d0}^{(0)} + a_{d0}^{(1)}(X^0) \end{pmatrix} A_d e^{-(\mathbf{X}/L_M)^2} e^{-iE_0 X^0} \tag{3.1}$$

$$\chi^{a\dot{e}}(x) = \delta^{a\dot{e}}\chi_0(x) = \delta^{a\dot{e}} e^{-(x^0/\tau_0)^2} e^{i\omega_0 x^0} \chi_0(r) = -\psi^{a\dot{e}}(x) \tag{3.2}$$

modifying (2.7) and (2.8). L_M^3 is a large volume in which the meson is likely to be found; no generality is lost by choosing this region to be centered at $\mathbf{X} = 0$. τ_0 is a long period of time into which the relative times of the quarks are likely to fall. Deviations introduced by the approximations (3.1) and (3.2) includes generation of triplet wave functions for vector mesons according to (I 6.4), where $\mathbf{K}$ is interpreted as $-i\,\partial/\partial\mathbf{X}$. These vanish in the limits of L_M, $\tau_0 \to \infty$. Integrals over $\mathbf{X}$ and x^0 in (2.4) are now finite due to the factor

$$\exp[-2(\mathbf{X}/L_M)^2 - 2(x^0/\tau_0)^2] \tag{3.3}$$

introduced.

$a_{d+}^{(0)}$ and $a_{d0}^{(0)}$ are unity associated with K^+ and K^0, respectively. They are introduced for generalization together with $\exp(-iE_0X^0)$ to annihilation operators, considered in Appendix B, later in Section 4. $a_{d+}^{(1)}(X^0)$ and $a_{d0}^{(1)}(X^0)$ are first-order quantities varying slowly in time and characterizing kaon decay, just like the similar quantities employed in nonrelativistic quantum mechanics.

4. DECAY AMPLITUDE

Convert all the products of first-order derivatives in (2.4) into a form like

$$(\partial\Psi_d^+\chi_a^{\dot{e}})(\partial_x\Psi_d\chi_{\dot{a}}^{e}) = \partial\Psi_d^+\chi_a^{\dot{e}}\partial_x\Psi_d\chi_{\dot{a}}^{e} - \Psi_d^+\chi_a^{\dot{e}}\partial\partial_x\chi_{\dot{a}}^{e} \tag{4.1}$$

Terms of the type of the last term together with the $\phi_p - M_m^2$ terms in (2.4) vanish since these are just the terms of the basic meson equations (A6), after conversion by means of (A3), times common wave functions. Surface terms like the first term on the right of (4.1) vanish for the $a_d^{(0)}$ part of (3.1) due to the factor (3.3). For the time-dependent perturbation $a_d^{(1)}(X^0)$, the surface terms arising from all products of the type (4.1) in (2.4) yield the first-order terms

$$\begin{aligned}
&\int d^4X \int d^4x \, \frac{1}{4}\, \{[(1-a)\partial^{\dot{b}a} - \partial_x^{\dot{b}a}]\Psi_d^+(X)\chi_a^{\dot{e}}(x)(a\partial_{ef} + \partial_{x\dot{e}f})\Psi_d\chi_{\dot{b}}^{f}(x) \\
&\quad + (a\partial_{e\dot{f}} + \partial_{xe\dot{f}})\psi_d^+(X)\chi_b^{\dot{f}}(x)\,((1-a)\partial^{b\dot{a}} - \partial_x^{b\dot{a}})\psi_d(X)\chi_{\dot{a}}^{e} \\
&\quad + \chi \to \psi,\ a \to 1-a,\ \partial_x \to -\partial_x\} \\
&\quad = -i\frac{1}{2}E_0[a_{d+}^{(1)*}(X^0 \to \infty)a_{d+}^{(0)} + a_{d0}^{(1)*}(X^0 \to \infty)a_{d0}^{(0)}] \\
&\quad \times A_d^2 \int d^3\mathbf{X}\, e^{-2(\mathbf{X}/L_M)^2} \int d^4x\, \chi_0^2(r)e^{-2(x^0/\tau_0)^2}
\end{aligned} \tag{4.2}$$

At laboratory time $X^0 \to -\infty$, meson decay has not started, so that $a_{d+}^{(1)}(X^0 \to -\infty) \to 0$ in (4.2). As considered in Appendix B, the following elevations into annihilation operators are made

$$a_{d+}^{(0)}e^{-iE_0X^0} \to a_{d+}, \qquad a_{d0}^{(0)}e^{-iE_0X^0} \to a_{d0} \tag{4.3a}$$

These refer to a K^+ and a K^0, respectively, at rest. The initial $|i\rangle$ and final $\langle f|$ states are a K^+ meson at rest and vacuum, respectively, denoted by

$$|i\rangle = |K^+\rangle, \qquad \langle f| = \langle 0|, \qquad \langle 0|0\rangle = 1 \tag{4.3b}$$

Therefore,

$$a_{d+}|i\rangle = |0\rangle, \qquad a_{d0}|i\rangle = 0 \tag{4.3c}$$

Sandwiching to (4.2) between $\langle f|$ and $|i\rangle$ yields

$$-i\frac{1}{2}E_0 S_{fi} A_d^2 \sqrt{\frac{\pi}{2}}\frac{\pi}{2} L_M^3 \int d^4x\, \chi_0^2(r) e^{-2(x^0/\tau_0)^2} \tag{4.4a}$$

where

$$S_{fi} = \langle 0 | a_{d+}^{(1)*}(\infty) e^{iE_0\cdot\infty} a_{d+} | K^+\rangle = a_{d+}^{(1)*}(\infty) e^{iE_0\cdot\infty} \tag{4.4b}$$

is interpreted as the usual S-matrix element or decay amplitude.

The first-order term (4.4a) is to be balanced by the remaining first-order terms containing gW in (2.4). Here, the constant value S_{2ML}, (2.1), is balanced off by the zeroth-order terms of Section 2. The first-order terms in S_L, (2.3), vanish for plane wave solutions for the muon and the neutrino. Since the K^+ decay is a charged one, the neutral gauge fields W_3 and C in (2.3d) are dropped. Further, W^+ defined by (8.5) below is seen to couple to a_{d0} and such terms also drop out by (4.3c). Only terms containing W^- remain and these read

$$\int d^4X \int d^4x \frac{i}{8} g\sqrt{2}\Bigg(\{[(1-a)\partial^{\dot{b}a} - \partial_x^{\dot{b}a}]\Psi_{d0}^* \chi_a^{\dot{e}}(x)\} a W_{\dot{e}f}^- \Psi_{d+} \chi_{\dot{b}}^f(x)$$
$$- W^{-\dot{b}a}\Psi_{d0}^* \chi_a^{\dot{e}}(x)(1-a)[(a\partial_{\dot{e}f} + \partial_{x\dot{e}f})\Psi_{d+}\chi_{\dot{b}}^f(x)] + \begin{pmatrix} \chi \to \psi \\ \partial_x \to -\partial_x \\ a \to 1-a \end{pmatrix} + \text{c.c.}\Bigg) \tag{4.5}$$

$a_{d0}^{(0)*}$ in Ψ_{d0}^* remains unity, as was mentioned below (3.3), and operationally corresponds to $a_{d+}^{(1)*}(X^0 \to \infty)$ in (4.2). For Ψ_{d+}, (4.3a) is used, as in (4.2). Sandwiching (4.5) between $\langle f|$ and $|i\rangle$ and applying (4.3b), (4.3c) leads to

$$\frac{1}{\sqrt{2}} g A_d^2 \int d^4X\, E_0 W^{0-}(X) e^{iE_0X^0 - 2(\mathbf{X}/L_M)^2} \int d^4x\, \chi_0^2(r) e^{-2(x^0/\tau_0)^2} \tag{4.6}$$

The decay amplitude is obtained by equating (4.4a) to the negative of (4.6):

$$S_{fi} = -i\frac{4g}{\pi\sqrt{\pi}} L_M^{-3} \int d^4X\, W^{0-}(X) e^{iE_0X^0 - 2(\mathbf{X}/L_M)^2} \tag{4.7}$$

The right of (4.7) is an integral over the first order interaction Lagrangian density. By following the usual quantization procedure (see e.g. Chap. 6 of Lee (1981) where Coulomb gauge is chosen), it can be shown that this density is simply the negative of the corresponding first order interaction Hamiltonian density, noting that $W^{0\pm}$ is independent of X here. Therefore, (4.7) can be regarded as derivable from (2.4) in the conventional S-matrix formalism, apart from a constant factor.

5. SECOND-ORDER PERTURBATION AND DECAY RATE

The first-order treatment of Section 4 led to the decay amplitude (4.7) dependent upon $W^{0\pm}$, which will now be obtained from the second-order terms in (2.1) identified by means of (2.6). Variation of (2.1) with respect to $W^{+a\dot{b}}(X)$ yields

$$\frac{1}{2}\Box W^{-}_{a\dot{b}} - \frac{1}{4}\,\partial_{a\dot{b}}(\partial^{\dot{c}d}W^{-}_{\dot{c}d}) - g^2(\cdots) + \frac{1}{2\sqrt{2}}\,g\chi_{\mu\dot{b}}\chi_{\nu a}$$

$$+ \frac{e_w}{32}\,g^2 \int d^4x\; W^{-}_{\dot{e}f}[\Psi^{*}_{\dot{d}+}\chi^{\dot{e}}_{a}(x)\Psi_{d+}\chi^{f}_{\dot{b}}(X) + \chi \to \psi + \text{c.c.}] \qquad (5.1)$$

where $g^2(\cdots)$ denotes W cubed terms to order g^2. Making use of (3.2) and the upper of (3.1) and contracting (5.1) by $\delta^{a\dot{b}}$ leads to

$$\Box W^{0-} - (\partial/\partial X^0)(\partial^\alpha W^{-}_{\alpha}) - M^2_W W^{0-} e^{-2(X/L_M)^2}$$

$$= -\frac{1}{2\sqrt{2}}\,g[(x_{\nu1}(X)\chi_{\mu\dot{1}}(X) + \chi_{\nu2}(X)\chi_{\mu\dot{2}}(X)] \qquad (5.2a)$$

$$M^2_W = \frac{1}{4}\,e_\omega g^2 \int d^3x\; A^2_{\dot{d}}\chi^2_{\dot{0}}(r) \int dx^0\; e^{-2(x^0/\tau_0)^2}$$

$$= \pi\sqrt{2\pi}\; e_\omega g^2 \beta_{m0}\tau_0 = (80.22\ \text{GeV})^2 \qquad (5.2b)$$

where M_W is identified as the $W^\pm$ mass generated by K^+ and differs by a factor proportional to $\tau_0\beta_{m0}/A^2_{\dot{d}}$ from the M_W expression of (II 6.12b). This is due to the fact that (B1) enters in the total action S_{T2} of (II 6.6), as considered in Appendix B, and due to (3.2). The large value of M_W stems from an infinite integral over the relative time x^0 made finite by the introduction of a cutoff parameter τ_0 in (3.2).

The first two terms of (5.2a) are much smaller than the third one (Marciano and Sirlin, 1976) and are therefore dropped. This corresponds to the conventional Fermi point interaction approximation. Eliminating W^{0-} between (5.2a) and (4.7) yields

$$S_{fi} = -\frac{i2}{\pi\sqrt{2\pi}}\,g^2 M_W^{-2} L_M^{-3} \int d^4X\; e^{iE_0X^0}(\chi_{\nu1}\chi_{\mu\dot{1}} + \chi_{\nu2}\chi_{\mu\dot{2}}) \qquad (5.3)$$

from which the decay rate

$$\Gamma(K^+ \to \mu^+\nu_\mu) = \sum_{\text{final states}} \Big|\sum_{\nu\ \text{spin}} S_{fi}\Big|^2/T \qquad (5.4)$$

is evaluated. Here, T denotes a long time period during which decay takes place.

6. $K^+ \rightarrow \mu^+ + \nu_\mu$ DECAY RATE

The complex conjugates of the plane wave functions of the particles μ^- and ν_μ represent the antiparticles μ^+ and $\bar{\nu}_\mu$, respectively, and read

$$\begin{pmatrix} \chi_{\nu 1}^{(-)} \\ \chi_{\nu 2}^{(-)} \end{pmatrix} = \frac{1}{\sqrt{2V}} e^{-i\mathbf{p}_\nu \mathbf{X} - iE_\nu X^0} \left[\begin{pmatrix} 1 - \dfrac{p_{\nu 3}}{E_\nu} \\ \dfrac{-p_{\nu 1} + ip_{\nu 2}}{E_\nu} \end{pmatrix}, \begin{pmatrix} \dfrac{-p_{\nu 1} - ip_{\nu 2}}{E_\nu} \\ 1 + \dfrac{p_{\nu 3}}{E_\nu} \end{pmatrix} \right] \tag{6.1a}$$

$$\begin{pmatrix} \chi_{\mu 1}^{(+)} \\ \chi_{\mu 2}^{(+)} \end{pmatrix} = \frac{1}{\sqrt{2V}} e^{-i\mathbf{p}_\mu \mathbf{X} + iE_\mu X^0} \sqrt{1 + \frac{m_\mu}{E_\mu}} \left[\begin{pmatrix} 1 + \dfrac{p_{\mu 3}}{E_\mu + m_\mu} \\ \dfrac{p_{\mu 1} - ip_{\mu 2}}{E_\mu + m_\mu} \end{pmatrix}, \begin{pmatrix} \dfrac{p_{\mu 1} + ip_{\mu 2}}{E_\mu + m_\mu} \\ 1 - \dfrac{p_{\mu 3}}{E_\mu + m_\mu} \end{pmatrix} \right] \tag{6.1b}$$

The superscript $(-)$ denotes negative-energy solutions for $\bar{\nu}_\mu$, so that (6.1a) corresponds to the creation of a positive energy ν_μ propagating in the opposite direction. The superscript $(+)$ denotes positive-energy solutions and the complex conjugate of (6.1b) corresponds to the creation of a μ^+. Here V is the normalization volume of the leptons, $\mathbf{p}$ their momenta, and $E > 0$ their energies, and the indices 1, 2, 3 indicate the three spatial directions. There are two solutions, which are separated by commas in the brackets and associated with opposite spin orientations, in each of the four wave functions.

Calculation can be simplified by choosing $\mathbf{p}_\nu$ parallel to the third axis of $\mathbf{X}$ and introducing a compensation factor of $1/\sqrt{8}$. Summing over the spins leads to

$$\begin{aligned} &\sum_{\nu\ \text{spin}} (\chi_{\nu 1}\chi_{\mu 1} + \chi_{\nu 2}\chi_{\mu 2}) \\ &= \frac{1}{\sqrt{8}V} e^{i(\mathbf{p}_\mu - \mathbf{p}_\nu)\mathbf{X} - i(E_\mu + E_\nu)X^0} \sqrt{1 + \frac{m_\mu}{E_\mu}} \left(1 - \frac{|\mathbf{p}_\mu|}{E_\mu + m_\mu}\right) \end{aligned} \tag{6.2}$$

Combining (6.2) with (5.3) and carrying out the integrations yield

$$\begin{aligned} \sum_{\nu\ \text{spin}} S_{fi} = &-\frac{i}{2\pi\sqrt{\pi}} g^2 M_W^{-2} L_M^{-3} V^{-1} \sqrt{1 + \frac{m_\mu}{E_\mu}} \left(1 - \frac{|\mathbf{p}_\mu|}{E_\mu + m_\mu}\right) \\ &\times (2\pi)^4 \delta(\mathbf{p}_\mu - \mathbf{p}_\nu) \delta(E_0 - E_\nu - E_\mu) \end{aligned} \tag{6.3}$$

Employing the known relations

$$\sum_{\text{final states}} = 2(2\pi)^{-6}V^2 \int d^3\mathbf{p}_\mu \int d^3\mathbf{p}_\nu$$

$$[\delta(\mathbf{p}_\mu - \mathbf{p}_\nu)\delta(E_0 - E_\nu - E_\mu)]^2 = (2\pi)^{-4}\Omega T\delta(\mathbf{p}_\mu - \mathbf{p}_\nu)\delta(E_0 - E_\nu - E_\mu)$$

$$E_\mu^2 = m_\mu^2 + \mathbf{p}_\mu^2$$

$$G = (\sqrt{2}/8)g^2/M_w^2 \tag{6.4}$$

(5.4), and (6.3), following the conventional procedure (Källén, 1964), we obtain the decay rate

$$\Gamma(K^+ \to \mu^+\nu) = \frac{N_e G^2 m_\mu^2}{\pi^5 L_M^6 \beta_{m0} E_0}\left[1 - \left(\frac{m_\mu}{E_0}\right)^2\right]^2 \tag{6.5}$$

where E_0 is the K^+ mass and (7.4) below has been used for the decay volume Ω in (6.4).

7. THE DECAY VOLUME

The decay rate (6.5) is proportional to the decay volume Ω. In the conventional approach (Källén, 1964), Ω is canceled by the same volume in the squared normalized amplitude $|\Psi_{KG}|^2$ of a Klein–Gordon particle

$$\Psi_{KG} = (2E_0\Omega)^{-1/2}e^{-iE_0X^0 + i\mathbf{K}\mathbf{X}} \tag{7.1a}$$

$$\frac{\partial}{\partial X^0}\int d^3\mathbf{X}\, i\left[\Psi_{KG}^*\left(\frac{\partial}{\partial X^0}\right)\Psi_{KG} - \text{c.c.}\right] = \frac{\partial}{\partial X^0}\int d^3\mathbf{X}\,\frac{1}{\Omega} = 0 \tag{7.1b}$$

$$\int d^3\mathbf{X}\,\frac{1}{\Omega} = 1 \tag{7.1c}$$

where the first integral has been normalized to unity. In the $\Omega \to \infty$ limit, $\Psi_{KG} \to 0$. In the spinor strong interaction theory, however, the corresponding amplitude (2.11) cannot vanish due to the requirement of confinement.

The analog of (7.1b) in the spinor strong interaction theory has been largely derived in the Appendix of II, and proceeds as follows. Take the complex conjugate of (A6b) and multiply it by $\psi_{\dot{d}}^{c}$. Subtract the resulting equation from (A6a) multiplied by $\chi_{\dot{a}}^{\dot{e}}$. The result can be put in the form of (II A2),

$$\partial_{\mathrm{I}}^{\dot{b}a}\chi_{\dot{a}}^{\dot{e}}\partial_{\mathrm{II}\dot{e}f}\chi_{\dot{b}}^{f} - \partial_{\mathrm{II}}^{\dot{a}d}\psi_{\dot{d}}^{c}\partial_{\mathrm{I}\dot{b}c}\psi_{a}^{\dot{b}} = (\partial_{\mathrm{I}}^{\dot{b}a}\chi_{\dot{a}}^{\dot{e}})(\partial_{\mathrm{II}\dot{e}f}\chi_{\dot{b}}^{f}) - (\partial_{\mathrm{II}}^{\dot{a}d}\psi_{\dot{d}}^{c})(\partial_{\mathrm{I}\dot{b}c}\psi_{a}^{\dot{b}}) \tag{7.2}$$

We insert the zeroth-order solutions (A5) and (2.7)–(2.9) for K^+ into (7.2) and integrate over $\mathbf{x}$ to obtain the first of (II A3) and (II 2.11a). Integrating once more over $\mathbf{X}$ and multiplying by i produces the equivalent of (7.1b),

$$(\partial/\partial X^0)\int d^3\mathbf{X}\int d^3\mathbf{x}\; E_0 A_d^2 \chi_0^2(r) = \partial N_e/\partial X^0 = 0 \tag{7.3}$$

Unlike (7.1b), where the integral has been normalized to unity, the integral of (7.3) cannot be so normalized, but assumes a large, constant value

$$N_e = 8\pi\beta_{m0}E_0\Omega \tag{7.4}$$

where (7.1c) and (2.10b) have been consulted. $N_e/4\pi\Omega$ is simply N_{c0} of (II 2.11a).

8. INCLUSION OF INTERNAL COORDINATES

A full description of the meson wave functions includes the internal functions, as in (I 5.4). For the K doublet, they enter like

$$\begin{pmatrix}\Psi_{d+}(X)\\ \Psi_{d0}(X)\end{pmatrix} \to \begin{pmatrix}\Psi_{d+}(X)\xi_{d+}(z,u)\\ \Psi_{d0}(X)\xi_{d0}(z,u)\end{pmatrix} \tag{8.1}$$

where the internal functions

$$\begin{aligned}\xi_{d+}(z,u) &= \frac{1}{\sqrt{2}}(z^1u_3 - u^1z_3)\\ \xi_{d0}(z,u) &= \frac{1}{\sqrt{2}}(z^2u_3 - u^2z_3)\end{aligned} \tag{8.2}$$

are the normalized forms of (I 9.1b) for the K's. Here z and u are the internal coordinates of both quarks and obey the same orthonormal relations as those for unitary spinors,

$$z^pz_r = u^pu_r = \delta^p_r \tag{8.3}$$

with all other bilinear products vanishing. Here, p and r are the flavors of the quark and antiquark, respectively, with the indices 1, 2, and 3 referring to up, down, and strange quarks, respectively.

The generalized (8.1) calls for a corresponding generalization of the off-diagonal elements of (2.3d). Thus,

$$\frac{i}{2}g\begin{pmatrix}W_3 & \sqrt{2}\,W^+\\ \sqrt{2}\,W^- & -W_3\end{pmatrix}\begin{pmatrix}\Psi_{d+}\\ \Psi_{d0}\end{pmatrix} \to \frac{i}{2}g\begin{pmatrix}W_3 & \sqrt{2}\,W^+D_+\\ \sqrt{2}\,W^-D_- & -W_3\end{pmatrix}\begin{pmatrix}\Psi_{d+}\xi_{d+}\\ \Psi_{d0}\xi_{d0}\end{pmatrix} \tag{8.4}$$

where

$$\sqrt{2}\, W^{\pm}(X) = W_1(X) - iW_2(X) \tag{8.5}$$

$$D_+ = z^1\, \partial/\partial z^2 - z_2\, \partial/\partial z_1 + z \to u$$

$$D_- = z^2\, \partial/\partial z^1 - z_1\, \partial/\partial z_2 + z \to u \tag{8.6a}$$

$$D_+\xi_{d0} = \xi_{d+}, \qquad D_-\xi_{d+} = \xi_{d0}, \qquad D_+\xi_{d+} = D_-\xi_{d-} = 0 \tag{8.6b}$$

Applying (8.1) and (8.4) to (2.4) and (2.5) and making use of (8.2), (8.3), (8.5), and (8.6) does not change (2.4). Similarly, ϕ_p in (2.4) remains unchanged after application of (A5) and (8.1)–(8.3) to (A4).

The representation of internal properties of hadrons in terms of internal functions and internal operators such as those of (8.2) and (8.6) is principally the same as the conventional representation in terms of complex vectors and matrix operators, such as the $SU(3)$ matrices. However, that each quark has its own internal coordinate z or u allows for an internal symmetry required to classify mesons (I 9.2) and baryons (Hoh, 1994b).

Further, the above representation puts the internal operators producing the quark masses, hadron charges and hypercharges, and weak charges as eigenvalues on a formally equal footing as the operator ∂_x in relative space-time in (2.4) producing the relative energy and momenta between the quarks and also the operator ∂ in (2.3d) in laboratory space-time producing the total energy and momentum of the meson. The quark mass operator $m_{\mathrm{op}}(z, u)$, (I 9.3), leads to the quark mass term M_m in (2.4), the hadron charge operator has been employed in Hoh (1994c), and the hadron hypercharge operator can be similarly defined and has the eigenvalue Y in (2.3d). The charged weak charge operators can be extracted from (8.4):

$$\tfrac{1}{2}g\sigma_{\pm} \to \tfrac{1}{2}g\sigma_{\pm}D_{\pm} \tag{8.7a}$$

$$\sigma_{\pm} = \frac{1}{\sqrt{2}}(\sigma_1 \pm i\sigma_2), \qquad \sigma_+ = \sqrt{2}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \sigma_- = \sqrt{2}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{8.7b}$$

9. ACTION FOR ISOTRIPLET MESON DECAY

The treatment of $\pi^+ \to \mu^+ + \nu_\mu$ decay below is entirely analogous to that for K^+ decay. The action S_{T3} of (II 7.6) is the isotriplet version of S_{T2} in Section 2. Augmenting it by the lepton actions S_L of (2.3) and observing Appendix B regarding (B1) leads to the total action

$$S_{3ML} = S_{F3} + S_{Ll} + S_{Lr} + S_{Lm} + S_{M3} \tag{9.1}$$

$$S_{F3} = -\frac{1}{4}\int d^4X \sum_{l=1}^{3} G_l^{\alpha\beta} G_{l\alpha\beta} \tag{9.2a}$$

$$S_{M3} = S_{M2} \quad \text{with} \quad D \to D_t \quad \text{and} \quad \Psi_d \to \Psi_t \tag{9.2b}$$

S_{M3} differs from S_{M3} of (II 7.4) in that the Cartesian representation there is replaced by spherical one here. The analogs of (2.3d) and (2.5), observing the left-hand side of (8.4), are

$$D_t^{a\dot{b}} = \partial^{a\dot{b}} + \frac{i}{2} g \begin{pmatrix} W_3 & \sqrt{2}\, W^+ & 0 \\ \sqrt{2}\, W^- & 0 & \sqrt{2}\, W^+ \\ 0 & \sqrt{2}\, W^- & -W_3 \end{pmatrix}^{a\dot{b}} \tag{9.3a}$$

$$\Psi_t(X) = \begin{pmatrix} \Psi_{t+}(X) \\ \Psi_{t0}(X) \\ \Psi_{t-}(X) \end{pmatrix} \tag{9.3b}$$

$\Psi_{t\pm}(X)$ and $\Psi_{t0}(X)$ are the wave functions of $\pi^{\pm}$ and π^0, respectively.

The transition of the gW term in (2.3d) to that in (9.3a) passes through the following steps. For pions, the analog of (8.1) is

$$\begin{pmatrix} \Psi_{t+}(X) \\ \Psi_{t0}(X) \\ \Psi_{t-}(X) \end{pmatrix} \to \begin{pmatrix} \Psi_{t+}(X)\xi_{t+}(z, u) \\ \Psi_{t0}(X)\xi_{t0}(z, u) \\ \Psi_{t-}(X)\xi_{t-}(z, u) \end{pmatrix} \tag{9.4}$$

and of (8.2) is

$$\xi_{t+} = \frac{1}{\sqrt{2}} (z^1 u_2 - u^1 z_2), \qquad \xi_{t-} = \frac{1}{\sqrt{2}} (z^2 u_1 - u^2 z_1)$$

$$\xi_{t0} = \tfrac{1}{2}(z^1 u_1 - z^2 u_2 - z_1 u^1 + z_2 u^2) \tag{9.5a}$$

The stepping operations corresponding to (8.6b) are

$$D_+\xi_{t-} = \sqrt{2}\, \xi_{t0}, \qquad D_+\xi_{t0} = \sqrt{2}\, \xi_{t+}, \qquad D_+\xi_{t+} = 0$$

$$D_-\xi_{t+} = \sqrt{2}\, \xi_{t0}, \qquad D_-\xi_{t0} = \sqrt{2}\, \xi_{t-}, \qquad D_-\xi_- = 0 \tag{9.5b}$$

The $SO(3)$ operators corresponding to $\sigma_\pm$ of (8.7b) are

$$S_+ = \frac{1}{\sqrt{2}} (S_1 + iS_2) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$S_- = \frac{1}{\sqrt{2}} (S_1 - iS_2) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{9.6}$$

Transition from the $SU(2)$ case of (8.7) to the $SO(3)$ case is just the replacement of σ by S on the left-hand side of (8.7a) together with the replacement

$$\frac{1}{2}g\sigma_3 W_3 = \frac{1}{2}gW_3\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \to \frac{1}{2}gS_3W_3 = \frac{1}{2}gW_3\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix} \quad (9.7)$$

which clearly shows the physical equivalence of both operators. Following up this transition by a generalization analogous to (8.7a) leads to

$$\tfrac{1}{2}g\sigma_\pm W^\pm \to \tfrac{1}{2}gS_\pm W^\pm \to \tfrac{1}{2}gS_\pm D_\pm W^\pm \quad (9.8)$$

Application of (9.3b)–(9.5) to (9.2b), observing (9.8) and (8.3), shows that (9.2b) remains unchanged and that (9.3a) holds; the formal equivalence of the gW operators in (9.3a) and those in (8.4) is manifest.

Note that the internal operators here are nontrivial since they produce the $\sqrt{2}$ factor in (9.5b) and hence in (9.3a). In the absence of internal functions, generalization takes place via $\boldsymbol{\sigma}\mathbf{W} \to \mathbf{SW}$ and the $\sqrt{2}$ factor in (9.3a) would be absent.

10. $\pi^+ \to \mu^+ + \nu_\mu$ DECAY

The treatment starts from (9.1) and follows the same procedures as those starting from (2.1). Thus, Appendix A and (2.6)–(2.11) hold with the subscripts $d \to t$ and the addition of the subscript $t-$ for π^-. All the Ψ's in (9.3b) have the same amplitude as those in (A5), so that the potential ϕ_p for and wave functions of K and π are the same, in accordance with Section 10 of I.

The ansatz (3.1) is replaced by

$$\begin{pmatrix}\Psi_{t+}\\ \Psi_{t0}\\ \Psi_{t-}\end{pmatrix} = \begin{pmatrix}a_{t+}^{(0)} + a_{t+}^{(1)}(X^0)\\ a_{t0}^{(0)} + a_{t0}^{(1)}(X^0)\\ a_{t-}^{(0)} + a_{t-}^{(1)}(X^0)\end{pmatrix} A_t e^{-(\mathbf{X}/L_M)^2 - iE_0X^0} \quad (10.1)$$

where $A_t = A_d$. The development from (3.2) to (4.7) similarly holds with the above replacements and with

$$|i\rangle = |\pi^+\rangle \quad (10.2)$$

$$a_{t+}|i\rangle = |0\rangle, \qquad a_{t0}|i\rangle = a_{t-}|i\rangle = 0 \quad (10.3)$$

instead of (4.3c). Sections 5–7 analogously hold and (5.2b) is recovered; π^+ generates the same $W^\pm$ mass as K^+ does. This is analogous to the fact that

K^0 and π^0 generate the same $W^\pm$ mass in (II 6.12b) and Section 7 of II. Thus, the decay rate (6.5) applies upon setting E_0 to the pion mass m_π,

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) = \frac{N_e G^2 m_\mu^2}{\pi^5 L_M^6 \beta_{m0} m_\pi} \left[1 - \left(\frac{m_\mu}{m_\pi}\right)^2\right]^2 \tag{10.4}$$

11. COMPARISON WITH DATA

The main prediction that can be checked with data is the ratio of K^+ decay, (6.5), to π^+ decay, (10.4). Putting $E_0 = m_K$, the kaon mass, in (6.5), we obtain from these relations

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \left(\frac{m_\pi}{m_K}\right)^5 \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.41 \tag{11.1}$$

which compares favorably with the observed value of 1.34 evaluated from data (Particle Data Group, 1994), from which the masses in (11.1) have also been obtained. Another form of comparison starts with the decay rates given in the literature (Lee, 1981),

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) = (F \cos \vartheta_C)^2 \frac{G^2 m_\mu^2}{2\pi m_\pi^3} (m_\pi^2 - m_\mu^2)^2 \tag{11.2}$$

$$\Gamma(K^+ \to \mu^+\nu_\mu) = (F \sin \vartheta_C)^2 \frac{G^2 m_\mu^2}{2\pi m_K^3} (m_K^2 - m_\mu^2)^2 \tag{11.3}$$

where F is called the charged pion and kaon decay constant. Comparison with (6.5) and (10.4) yields the Cabbibo angle via

$$\tan \vartheta_C = m_\pi / m_K = 0.2827 \tag{11.4}$$

Compared to the observed value of 0.275, the agreement may be considered as good in view of the fact that the muon mass term S_{Lm} in (2.3) breaks the $SU(2)$ symmetry and that radiative corrections to these processes are fairly large.

Decay of the K and π into positron and neutrino is found by replacing the muon mass by the electron mass. The ratios of these decays are the same as those in the literature (Källén, 1964; Lee, 1981). The D and $B \to \mu + \nu_\mu$ decay rates are obtained by replacing the kaon mass by the D and B masses, respectively. These rates are essentially in inverse proportion to their masses and are too small to be observed, in agreement with data (Particle Data Group, 1994).

Comparison of (10.4) to (11.2) yields

$$(F \cos \vartheta_C)^2 = \frac{8N_e}{\pi^4 m_\pi^2 \beta_{m0} L_M^6} = \frac{64}{\pi^3 m_\pi} \frac{\Omega}{L_M^6} \tag{11.5}$$

where (7.4) has been used. Thus, the decay constant F is essentially the ratio of two large volume-related constants, $\sqrt{\Omega}$ and L_M^3, which render two infinite integrals over $\mathbf{X}$ finite. Conversely, (11.5) shows that the decay volume Ω is proportional to the square of the meson volume L_M^3. With the data $F \cdot \cos \vartheta_c = 0.128$ Gev, this proportionality constant is

$$L_{M\Omega}^{-3} = \Omega/L_M^6 = (0.165 \text{ Gev})^3 \tag{11.6}$$

which is also proportional to $|\psi(0)|^2$, the square of the conventional quark-antiquark wave function at origin (van Royen and Weiskopf 1967).

If L_M^6, Ω, $\tau_0 \to \infty$ were allowed, the zeroth-order solutions (2.7) and (2.8) would be recovered. However, (5.2b) shows that the relative time scale τ_0 is limited by M_W. The β_{m0} of (2.10b) has been obtained via numerical integrations in I and Hoh (1996) and ranges from 0 for no confinement to 0.024 GeV^3 for what appears to be a moderate confinement with a strong interaction meson radius less than 1 fm. Using this value and putting the relative energy ω_0 to 0 in the second of (2.9), we obtain from (5.2b) with $g^2 = 0.03187$ a value of $\tau_0 \sim 1.3 \times 10^6$ fm.

This τ_0 far exceeds the above meson size. According to the consideration from (B10b) to (B11), L_M is estimated to be of the same order as τ_0, or $L_M \sim 2 \times 10^6$ fm. Inserting these values and m_π into (11.5) yields a scale $\Omega^{1/3} \sim 3.3 \times 10^{11}$ fm for the pion. The corresponding value for the kaon is $(m_\pi/m_K)^{1/3}$ times smaller. These values show that the approximate Ansätze (3.1), (3.2), and (10.1) are fairly close to the zeroth-order solutions (2.7) and (2.8) and are therefore justifiable.

12. PURELY LEPTONIC INTERACTIONS AND NATURE OF $W^\pm$ AND Z

It will be shown below that the purely leptonic interaction results of the standard electroweak model, such as decay of muon at rest, can be taken over here. Nonleptonic interactions are then briefly discussed.

The lepton action of the muon family of (2.3) is generalized to include the electron and taon families, so that (2.1) becomes

$$S_{2ML} = \int d^4X\, \mathscr{L}_{2ML} = S_{F2} + \sum_{\mu \to e,\mu,\tau} (S_{Lr} + S_{Ll} + S_{Lm}) + S_{M2} \tag{12.1}$$

where e denotes the electron and τ the taon.

The corresponding action in the standard electroweak model without quarks [(22.58) of Lee (1981)] reads

$$S_{SM} = \int d^4X\,\mathcal{L}_{SM} = S_{F2} + \sum_{\mu\to e,\mu,\tau} (S_{Lr} + S_{Ll} + S_{LmH}) + S_{H2} \tag{12.2a}$$

$$S_{H2} = \int d^4X \left[\frac{1}{4}(\sigma_\alpha^{a\dot{b}} D_{\dot{b}a}\phi_H)^+(\sigma^{\alpha a\dot{b}} D_{\dot{b}a}\phi_H) - U(\phi_H)\right. \tag{12.2b}$$

$$U(\phi_H) = \frac{\mu_H^2}{4\rho_H^2}(\phi_H^+\phi_H - \rho_H^2)^2 \tag{12.2c}$$

$$\phi_H = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \tag{12.2d}$$

where S_{H2} is the action for the Higgs field ϕ_H together with its coupling to the gauge fields. U is a potential and μ_H and ρ_H are constants.

S_{LmH} is the lepton mass term and is an integral over trilinear products of the right- and left-handed lepton fields and the Higgs field. It is invariant under $SU(2) \times U(1)$ transformations, contrary to its counterpart S_{LM} of (2.3c). It is not possible to modify S_{LM} to a scalar like S_{LmH} because ϕ_H entering the latter is a scalar, while its counterpart, the kaon, is the time component of a vector in the rest frame. If the kaon doublet is replaced by a pion triplet, it is not even possible to form a one-component quantity like S_{Lm}. However, this discrepancy can be overlooked here, since the lepton mass terms are not directly involved in obtaining the results below; a Glashow model has been adopted here.

Equations (2.1) and (12.2a) now differ only in their last terms S_{M2} and S_{H2}, which are compared below. After suitable transformations, (12.2d) is usually put in the form

$$\phi_H = \begin{pmatrix}0\\ \rho_H + \chi_H(X)\end{pmatrix} \tag{12.3}$$

where $\chi_H(X)$ is the Higgs field with mass μ_H. In S_{H2}, terms quadratic in the $W^\pm$ and Z, the massive neutral gauge field consisting of the usual linear combination of W^3 and C (Lee, 1981), provides masses to these fields. Upon variation with respect to $W^{\alpha+}$, the leading terms of S_{F2} and S_{H2} yield

$$\Box W_\alpha^- - \partial_\alpha\partial^\beta W_\beta^- + M_{WH}^2 W_\alpha^- = g^2(\cdots)_\alpha \tag{12.4a}$$

$$M_{WH} = g\rho_H/\sqrt{2} \tag{12.4b}$$

where $g^2(\cdots)_\alpha$ is given by (C3) in Appendix C.

In S_{M2}, the role of the Higgs field $\chi_H(X)$ is played by the zeroth-order solutions (2.7) and (2.8) or rather (3.1) and (3.2). The counterpart of (12.4a) is obtainable from (5.1) with (3.2) and (5.2b) and its form has been given by (II 6.13a),

$$\Box W_\alpha^- - \partial_\alpha \partial^\beta W_\beta^- - M_W^2 W^{-\alpha} = g^2(\cdots)_\alpha \tag{12.5}$$

Putting $M_{WH} = M_W$, we can determine the constant ρ_H.

Purely leptonic interactions involve 4 of the 12 leptons inside the summation sign in (12.1) or (12.2a). There is thus an extra set of lepton actions in addition to that in (2.1). If the massive lepton in this set is decaying, its wave functions are considered to be of zeroth order in (2.6).

There are four gauge boson types mediating purely leptonic interactions. The gauge bosons may be charged or neutral, i.e., $W^{\pm}$ or Z, A, defined in Appendix C. They may also be singlet or triplet, associated with a pair of leptons with their spins antiparallel or parallel, respectively, at the interaction vertex. These four types are: (i) $W_{\bar{0}}^{\pm}$, (ii) $\mathbf{W}^{\pm}$, (iii) Z_0, A_0, and (iv) $\mathbf{Z}$, $\mathbf{A}$.

It is seen that (12.4a) and (12.5) are the same for type (ii). If neutral processes are considered, $\mathbf{W}^{\pm}$ in these equations is simply replaced by $\mathbf{Z}$, $\mathbf{A}$ and the same conclusion applies also to type (iv). In Appendix C, it is shown that (12.4a) and (12.5) also produce the same observable results for types (i) and (iii).

Therefore, the Higgs boson can be replaced by the K, D, and B isodoublets for all four types. It can also be replaced by the π isotriplet if no Z is involved.

The Higgs potential (12.2c) is chosen ad hoc such that the amplitude $|\phi_H| = \rho_H \neq 0$ in the ground state in order to generate nonzero M_{WH} in (12.4b). No such assumption is needed in S_{M2}, where the corresponding amplitude of (2.7), (2.8), (3.1), and (3.2) are necessarily finite due to the requirement of confinement, as is shown by (2.11) and mentioned below (7.1).

It may noted that for type (ii), (C3c) vanishes for plane wave $\mathbf{W}^{\pm}$ fields with fixed polarizations. The same holds for type (iv) if $\mathbf{W}^{\pm}$ is replaced by $\mathbf{Z}$, $\mathbf{A}$. Thus nonlinear dispersion of the gauge boson fields due to their non-Abelian nature drops out here. The gauge boson fields here obey a Proca equation with a source provided by lepton fields.

While the standard model has been quite successful for purely leptonic interactions, it has not been so when quarks are included via Cabbibo-type theories. The quarks so introduced, as also in QCD and chiral symmetry considerations, are represented by Dirac wave functions. According to the spinor strong interaction theory of I, such representations are incorrect, since quarks have not been observed, contrary to leptons, which are described by Dirac wave functions.

This obvious and fundamental observation forms the basis for the construction of the spinor strong interaction theory. For semileptonic and nonleptonic interactions, the action of the standard model including quarks [(22.114) of Lee (1981)] is replaced by S_{2ML} and S_{3ML} here.

In nonleptonic interactions, two or more mesons are in motion. Equation (I 6.4) shows that the simple rest-frame description (A7) and (A8) is lost and the full (A6) and (A4) have to be dealt with. This appears to be highly complicated, involving higher order partial differential equations. The situation is analogous to that of treating positronium in motion with the Bethe–Salpeter equation.

This circumstance prevents a straightforward solution of the seemingly simple $K^{\pm} \to \pi^{+} + \pi^{0}$ decay by means of the candidate action S_{2ML} complemented by S_{M3}. For $K^{+} \to 3\pi$, however, the pions move nonrelativistically and (I 6.4) and its companion equation can, in principle, be solved perturbatively in powers of the pion momenta.

APPENDIX A. REVIEW OF BASIC MESON EQUATIONS

The spinors are related to Dirac bispinors ψ via (I A3),

$$\chi_1 = \psi_1 + \psi_3, \qquad \chi_{\dot{1}} = \psi_2 + \psi_4$$
$$\psi^1 = \psi_1 + \psi_3, \qquad \psi^2 = \psi_2 - \psi_4 \tag{A1}$$

Further,

$$\sigma_\alpha^{ab} X^\alpha = \delta^{ab} X^0 + \boldsymbol{\sigma}^{ab}\mathbf{X} = \begin{pmatrix} X^0 + X^3 & X^1 - iX^2 \\ X^1 + iX^2 & X^0 - X^3 \end{pmatrix} \tag{A2}$$

(I 6.2) defines

$$x^\alpha = x_{\mathrm{II}}^\alpha - x_{\mathrm{I}}^\alpha, \qquad X^\alpha = (1 - a)x_{\mathrm{I}}^\alpha + a x_{\mathrm{II}}^\alpha \tag{A3}$$

where X is the laboratory coordinate of the meson, x the relative coordinate of the quark coordinates x_{I} and x_{II}, and a denotes a constant.

In (2.4), $\Psi_d(X)$ is the meson wave function in laboratory space-time, $\chi_{\dot a}^{e}(x)$ and $\psi_{\dot a}^{e}(x)$ are the meson wave functions in relative space-time, $\chi_a^{\dot e}(x)$ and $\psi_a^{\dot e}(x)$ are their complex conjugates, M_m is the average quark mass of (I 9.6b), and $\phi_p(x)$ is the interquark potential determined by (I 4.12) or

$$\Box_{\mathrm{I}}\Box_{\mathrm{II}}\phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) = \tfrac{1}{2}\,\mathrm{Re}[\Psi_d^{\dagger}(X)\Psi_d(X)\psi_b^{g}(x)\chi_a^{b}(x)] \tag{A4}$$

with $\Psi_d \propto \exp[i\varphi_M(X)]$, (II 4.8), so that the right-hand side, hence ϕ_p, depends only upon x. The $g_A g_B$ factor in (I 4.12) has been absorbed into χ and ψ, as in (I 6.11).

A restricted type of variation of (2.4) with respect to the Hermitian conjugate of

$$\chi_{\dot{a}}^{e}(x_{\mathrm{I}}, x_{\mathrm{II}}) = \begin{pmatrix} \Psi_{d+}(X) \\ \Psi_{d0}(X) \end{pmatrix} \chi_{\dot{a}}^{e}(x) \tag{A5}$$

and a similar relation for ψ yield the basic meson equations

$$\partial_{\mathrm{I}}^{ab}\partial_{\mathrm{II}f\dot{e}}\chi_{\dot{b}}^{f}(x_{\mathrm{I}}, x_{\mathrm{II}}) = [\phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) - M_m^2]\psi_{\dot{e}}^{a}(x_{\mathrm{I}}, x_{\mathrm{II}}) \tag{A6a}$$

$$\partial_{\mathrm{I}\dot{c}b}\partial_{\mathrm{II}}^{\dot{d}\dot{e}}\psi_{\dot{e}}^{b}(x_{\mathrm{I}}, x_{\mathrm{II}}) = [\phi_p(x_{\mathrm{I}}, x_{\mathrm{II}}) - M_m^2]\chi_{\dot{c}}^{d}(x_{\mathrm{I}}, x_{\mathrm{II}}) \tag{A6b}$$

according to Section 3 of II.

For ground-state pseudoscalar mesons at rest, the solution (2.7) and (2.8) leads to the eigenvalue equation

$$[E_0^2/4 - M_m^2 + \phi_p(r) + \partial^2/\partial r^2 + 2\,\partial/\partial r]\chi_0(r) = 0 \tag{A7}$$

together with the solution of (A4)

$$\phi_p(r) = -\phi_p'(r) + d_m/r + \phi_0 \tag{A8}$$

according to Section 7 of I. Here ϕ_p' is the confining potential, which for $r \to \infty$ leads to (2.10); d_m and ϕ_0 are integration constants (Hoh, 1996).

APPENDIX B. QUANTIZATION AND HAMILTONIAN

The quantization procedure and the Hamiltonian in the rest frame have been considered in Sections 8 and 9 of II. The treatment, however, contains some inconsistencies, although these do not affect the main features of the results there. In the first place, S_{M2} and S_{M3} in (2.1) and (9.1) have been multiplied by

$$\mathcal{H}_M = \left[-4\int d^4x\, A_d^2\chi_0^2(r)\right]^{-1} \tag{B1}$$

of (II 4.7a) in S_{T2}, (II 6.6), and S_{T3}, (II 7.6), in order to remove the infinity arising from integration over the relative time x^0. Variation of $\mathcal{H}_M S_{M2}$ instead of S_{M2} will, however, not lead to (A6), due to the appearance of meson wave functions in both S_{M2} and (B1). Second, a Klein–Gordon type of amplitude expansion in terms of (7.1a) has been adopted in (II 8.5). This amplitude vanishes when the volume Ω approaches infinity and is inconsistent with (2.11) or Section 7, where a finite amplitude is required for confinement. Third, the annihilation and creation operators defined in (II 8.4), (II 8.5), (II 8.7) differ from the conventional ones in that they contain e_w of (2.9), so that the commutation relations (II 9.1), (II 9.2) and Hamiltonian (II 9.5) also depend upon e_w. In the $\omega_0 = 0$ limit, however, the conventional expressions are recovered.

These inconsistencies are removed here. The first one has been removed in (2.1) and (9.1). The remaining two are removed by an expression akin to (3.1),

$$\Psi = A_d e^{-(\mathbf{X}/L_M)^2}(a_{d(+)}e^{-iE_0X^0} + a^*_{d(-)}e^{iE_0X^0}) \tag{B2}$$

Here, only mesons at rest are considered. The Lagrangian is the integral over x in (2.4), from which the canonical momentum conjugate to (B2) is obtained by analogy to (II 8.1)–(II 8.3) or

$$\Pi = b_0 A_d e^{-(\mathbf{X}/L_M)^2} iE_0(a^*_{d(+)}e^{-iE_0X^0} + a_{d(-)}e^{-iE_0X^0}) \tag{B3}$$

where

$$b_0 = \frac{1}{2}\int d^4x \, |\chi_0(X)|^2 \tag{B4}$$

and restriction to pseudoscalar mesons according to (3.2) has been made. The complex conjugates of Ψ and Π are denoted by Ψ^+ and Π^+, respectively. In obtaining (B4), it has been tacitly assumed that ω_0 in (3.2) associated with the $a_{d(-)}$ term changes sign just like E_0 does relative to that of the $a_{d(+)}$ term, so that particle–antiparticle symmetry is maintained.

Consider the conventional quantum condition (II 8.9)

$$[\Pi(X^0, \mathbf{X}'), \Psi(X^0, \mathbf{X})] = [\Pi^+(X^0, \mathbf{X}'), \Psi(X^0, \mathbf{X})] = -i\delta(\mathbf{X} - \mathbf{X}') \tag{B5}$$

with all other commutators vanishing, and integrate it over $\mathbf{X}'$. For the (7.1a) type of wave functions employed in (II 8.5) satisfying (7.1c), each of the product terms on the left-hand side of (B5) contributes a value with magnitude 1/2. For (B2) and (B3) types of functions, the corresponding magnitude is

$$N_f/2 = \frac{E_0}{2}A_d^2 \int d^3\mathbf{X}\, e^{-2(\mathbf{X}/L_M)^2} \int d^4x \, |\chi_0(x)|^2 \tag{B6}$$

$N_f >> 1$ is an integration constant analogous to the first integral of (7.1b) and to (7.4). Instead of the zeroth-order solutions (2.7) and (2.8) used to obtain N_e in (7.4), the Ansatz (3.1) and (3.2) is inserted into (7.2), which upon integration over $\mathbf{X}$ and x leads to $\partial N_f/\partial X^0 = 0$.

This suggests that the commutators in (B5) have to be normalized by $N_f/2$,

$$[\Pi(X^0, \mathbf{X}), \Psi(X^0, \mathbf{X}')] = [\Pi^+(X^0, \mathbf{X}), \Psi(X^0, \mathbf{X}')] = -iN_f\delta(\mathbf{X} - \mathbf{X}') \tag{B7}$$

Applying quantized operators of the type (4.3a) to (B2) and (B3) together with their complex conjugates and employing (B7) leads to the usual commutation relations and pseudoscalar mesons at rest

$$[a_{d(+)}, a^*_{d(+)}] = [a_{d(-)}, a^*_{d(-)}] = 1 \tag{B8}$$

with all other commutators vanishing.

The Hamiltonian (II 9.3) is similarly normalized and becomes

$$H = \frac{1}{N_f}\int d^3\mathbf{X}\left[\frac{1}{b_0}(\Pi\Pi^+ + \Pi^+\Pi) - \mathcal{L}_{M2}\right] \tag{B9}$$

The same wave functions used to compute N_f are employed to evaluate the last term of (B9). Making use of (B3), (A7), and the boundary condition $\chi_0(r \to \infty) = 0$, we find that (B9) becomes

$$H = E_0(a^*_{d(+)}a_{d(+)} + a^*_{d(-)}a_{d(-)} + 1 + \Delta_R) \tag{B10a}$$

which is the usual expression, apart from Δ_R, which arises in the $\mathcal{L}_{M2}$ term due to the approximations introduced into (3.1) and (3.2) relative to (2.7) and (2.8),

$$\Delta_R = 2E_0^{-2}(\tau_0^{-2} - 3L_M^{-2} - \Delta\phi_p - \Delta(\chi)) \tag{B10b}$$

$\Delta\phi_p$ represents an average of the deviation of $\phi_p(r)$ in (A8) from the actual ϕ_p determined by (A4), and $\Delta(\chi)$ is an integral over the triplet meson wave function perturbations mentioned above (3.3). Both terms arise from the last-mentioned approximations and are not readily computed. However, (B10b) indicates that they are of the order of L_M^{-2} on dimensional grounds.

For H to reproduce the correct rest-energy eigenvalue E_0,

$$\Delta_R = 0 \tag{B11}$$

is required.

APPENDIX C. TYPES (i) AND (iii) IN SECTION 12

The physical gauge bosons are defined in terms of the gauge fields in (2.3d) and read

$$\sqrt{2}\, W^{\pm}_{\dot{b}a} = W_{1\dot{b}a} \mp iW_{2\dot{b}a}, \qquad g_0 Z_{\dot{b}a} = gW_{3\dot{b}a} - g'C_{\dot{b}a}$$

$$g_0 A_{\dot{b}a} = g'W_{3\dot{b}a} + gC_{\dot{b}a}, \qquad g_0^2 = g^2 + g'^2 \tag{C1}$$

For type (i), (12.4a) and (12.5) lead to

$$-(\partial^2/\partial\mathbf{X}^2)W_0^- + M^2_{WH}W_0^- = -g^2(\cdots)_0 \tag{C2a}$$

$$-(\partial^2/\partial\mathbf{X}^2)W_0^- - M^2_W W_0^- = -g^2(\cdots)_0 \tag{C2b}$$

respectively. The last term can be evaluated from a variation of (2.2):

$$g^2(\cdots)_\alpha = (g^2(\cdots)_0, \mathbf{g}^2(\cdots)) \tag{C3a}$$

$$g^2(\cdots)_0 = g^2[(\mathbf{W}^-)^2 W_0^+ - (\mathbf{W}^-\mathbf{W}^+)W_0^-] \tag{C3b}$$

$$\mathbf{g}^2(\cdots) = g^2[W_0^- W_0^+ \mathbf{W}^- - (W_0^-)^2\mathbf{W}^+ + (\mathbf{W}^-)^2\mathbf{W}^+ - (\mathbf{W}^-\mathbf{W}^+)\mathbf{W}^-] \tag{C3c}$$

For type (i), (C3b) and the right-hand side of (C2) vanish. The first term of (C2a) and (C2b) represents the square of momentum transfer between the leptons at the W_0^- vertex and is usually negligible next to the second term there due the large M_W^2 value. Therefore, (C2a) and (C2b) differ only in the sign of the M_W^2 term. This sign difference, however, does not alter the observable quantities, which are proportional to $(\pm M_W^2)^{-2}$.

Therefore, (C2a) and (C2b), hence also (12.4a) and (12.5), are the same for type (i). By replacing $W_0^\mp$ by Z_0 and M_W by M_Z, the above result for type (i) can be taken over for type (iii). In this result, it is noted that replacement of $W_0^\mp$ by A_0 and M_W by $M_A = 0$ leads to (C2a) and (C2b) becoming the same.

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